# Technical Notes

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## A Statistical Analysis of the Roll Rate of a Launch Vehicle under the Influence of Random Fin Misalignments

RAGAN BURT MADDEN*
*NASA Langley Research Center, Hampton, Va.*

### Nomenclature

$a$ = maximum value of fin misalinement or fin tolerance setting, radians
$C_{l_p}(t)$ = roll damping derivative, 1/rad
$C_{l_{\delta_1}}(t)$ = roll forcing derivative per fin of first stage, 1/rad
$d$ = reference length associated with $C_{l_p}$, ft. (m)
$d_1$ = reference length associated with $C_{l_{\delta_1}}$, ft. (m)
$F(p;t)$ = cumulative probability distribution function of roll rate $p$ at time $t$
$I(t)$ = moment of inertia along longitudinal axis, slug-ft$^2$ (kg-m$^2$)
$m_1$ = number of fins on first stage
$p(t)$ = roll rate, rad/sec.
$\dot{p}(t)$ = roll acceleration, rad/sec$^2$
$q(t)$ = dynamic pressure, lb/ft$^2$ (kg/m$^2$)
$R(t)$ = solution to a modified roll equation, rad/sec
$\dot{R}(t)$ = $dR(t)/dt$, rad/sec$^2$
$S$ = reference area associated with $C_{l_p}$, ft$^2$ (m$^2$)
$S_1$ = reference area associated with $C_{l_{\delta_1}}$, ft$^2$ (m$^2$)
$t$ = time, sec
$U(t)$ = freestream velocity, fps (m/sec)
$\delta_{1,j}$ = fin misalinement of $j$th fin on first stage, rad

### Introduction

BECAUSE of incorrect manufacture or attachment of the components comprising a launch vehicle, it will possess certain asymmetries such as body, thrust, or fin misalignments. These asymmetries can drastically affect the vehicle's desired performance and need to be considered in the preflight analysis.

The purpose of this Note is to present a method which statistically analyzes the effect of one of the preceding types of asymmetry—namely, fin misalignments—upon the roll rate of a vehicle. The problem is realistic in that the fin misalignments are random quantities, that is, their exact values are indeterminate within measurement limitations. The roll rate, being a function of the fin misalignment, will therefore be a random quantity and, consequently, is amenable to statistical analysis.

The importance of the problem lies in the fact that uncontrolled spin or roll can have serious consequences on performance and can produce catastrophic results. When the roll rate becomes coincident with the pitching frequency, a condition of pitch-roll resonance will occur and undesirable precession and nutation amplitudes (coning motion) can result. This motion can drastically reduce performance such as decreasing the altitude of apogee and, in some cases, can cause structural failure. It is a design objective to assure a highly improbable situation of having substantial periods of coincidence or near coincidence between the vehicle roll rate and the pitching frequency.

It is practically impossible for a finned vehicle to sustain a zero roll throughout the flight. The roll rate usually generates progressively over the period of flight in the sensible atmosphere. To aggravate the problem, the pitching frequency usually degenerates, and the two parameters tend to approach each other. One of the primary design objectives is to consider the statistical behavior of the roll rate and obtain some idea of the probability of pitch-roll resonance occurring. Figure 1 depicts a typical 0.99 roll dispersion curve and a pitching frequency curve. The 0.99 roll dispersion curve represents roll rates which will be exceeded with probability of 0.01 at each time. Since the curves intersect at $t_c$, the probability of pitch-roll resonance occurring is at least equal to 0.01.

### Fin Misalignments

Let $\delta_{i,j}$ represent the misalignment of the $j$th fin on the $i$th stage. The $\delta_{i,j}$ are assumed to be independent random variables having uniform probability density distributions with zero mean. For a given stage $i_0$, the $\delta_{i_0,j}$ are identically distributed; that is, they have the same range or tolerance. However, the range of the $\delta_{i,j}$ may vary from stage to stage.

The uniform distribution on the fin misalignments is considered realistic in view of the fabrication techniques. The machinist is usually instructed to establish a fin setting of a prescribed quantity with $\pm$ tolerances. The uniform distribution is one in which the random quantity is equally likely to occur any place within prescribed bounds.

### Method of Approach

The differential equation describing the roll rate of a symmetric, single-stage, finned vehicle is given by (suppressing the time parameter $t$)

$$I\dot{p} = \frac{C_{l_p} dS^2 qp}{2U} + \sum_{j=1}^{m_1} C_{l_{\delta_1}} \delta_{1,j} d_1 S_1 q \qquad (1)$$

with initial condition $p(0) = 0$. Considering only the magnitude of the roll rate, the solution can be written in the form

$$p(t) = |R(t)| \left|\sum \delta_{1,j}\right|$$

where $R(t)$ is the solution to a modified roll equation

$$I\dot{R} = (C_{l_p} dS^2 qR/2U) + C_{l_{\delta_1}} d_1 S_1 q$$

with initial condition $R(0) = 0$.

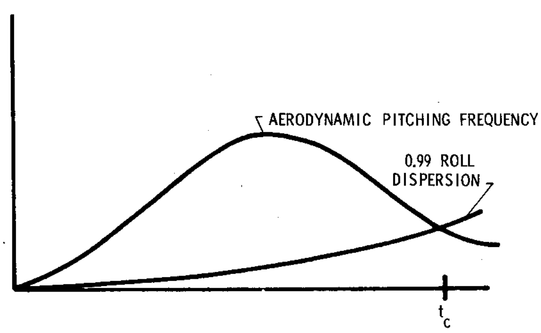


**Fig. 1 Typical pitching frequency and roll dispersions.**

Presented as Paper 70-1382 at the AIAA 2nd Sounding Rocket Technology Conference, Williamsburg, Va., December 7–9, 1970; submitted December 12, 1970; revision received November 8, 1971.

* Aero-Space Engineer, Fluid Analysis Section, Systems Engineering Division.

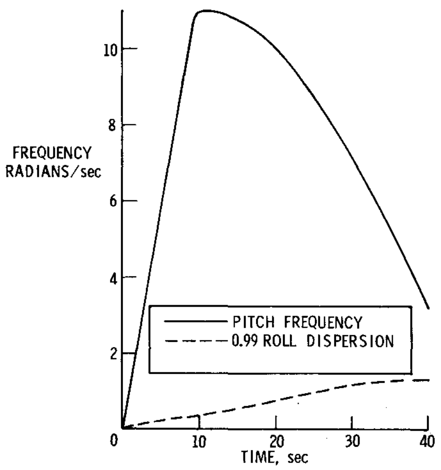


**Fig. 2 Pitch frequency and 0.99 roll dispersion for first 40 sec of SPED II flight.**

Using basic statistical techniques, the cumulative probability distribution function $F(p;t)$ of the roll rate $p$ at time $t$ is obtained. This function gives, at each time $t$, the probability that the roll rate is less than or equal to any value $p$. Closed-form solutions for $F(p;t)$ are obtained for three and four-finned, single-stage vehicles. The detailed derivations of $F(p;t)$ are given in Ref. 1. The results for a four-finned, single-stage vehicle are given below. We have

$$F(p;t) = 0 \qquad ;p \leqq 0$$

$$= \frac{1}{8a^4|R|}\left[\frac{p^4}{8|R|^3} - \frac{2ap^3}{3R^2} + \frac{16a^3p}{3}\right] \qquad ;0 \leqq p \leqq 2a|R|$$

$$= \frac{11}{12} + \frac{1}{8a^4|R|}\left[\frac{-p^4}{24|R|^3} + \frac{2ap^3}{3R^2} - \frac{4a^2p^2}{|R|} + \frac{32}{3}a^3p - 10a^4|R|\right] \qquad ;2a|R| \leqq p \leqq 4a|R|$$

$$= 1 \qquad ;p \geqq 4a|R|$$

For multistage vehicles with fins on each stage, closed-form solutions for $F(p;t)$ are not derived due to their complexity. However, by assuming the roll rate to be distributed as a normal random variable with known mean and standard deviation an approximate solution to $F(p;t)$ is obtained. This solution is given in Ref. 1.

Another type problem is also solved. This is the reverse of the problem described previously. That is, one may be given a maximum allowable roll rate which is to be exceeded with a prescribed probability. The problem is to determine the value of $a$ such that the resulting roll rate dispersions corresponding to the prescribed probability are less than the given maximum allowable roll rate. The solution to this problem is also given in Ref. 1.

### Applications

The NASA Langley Research Center SPED II launch vehicle was analyzed for the possibility of pitch-roll resonance occurring during the first 40 sec of flight. SPED II is a single-stage vehicle with four fins which are nominally set at zero cant angle. A fin misalinement of 0.00175 rad (0.1°) was allowed. The 0.99 roll dispersion curve was obtained by using the preceding equation and is shown in Fig. 2 along with the theoretical pitch frequency. Because of excess scatter, no reliable inflight roll rate data were available.

One observes that there is virtually no chance of pitch-roll resonance occurring during the first 40 sec of flight for the SPED II vehicle. Also, it was felt that a maximum allowable roll rate of 2 rad/sec which is exceeded with probability 0.01 would be acceptable. The maximum fin tolerance which yields 0.99 roll rate dispersions less than 2 rad/sec was found to be 0.00273 rad.

### References

[1] Madden, R. B., "A Statistical Analysis of the Roll Rate of a Launch Vehicle Under the Influence of Random Fin Misalinements," AIAA Paper 70-1382, Williamsburg, Va., 1970.

[2] Papoulis, A., "Probability, Random Variables, and Stochastic Processes," McGraw-Hill, New York, 1965.

[3] Parzen, E., "Modern Probability Theory and Its Applications," Wiley, New York, 1960.

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# Biot's Variational Principle for a Stefan Problem

ANANT PRASAD*
*Birla Institute of Technology and Science, Pilani, India*

AND

H. C. AGRAWAL†
*Indian Institute of Technology, Kanpur, India*

### Nomenclature

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| $c$ | = heat capacity per unit volume of the material |
| $F$ | = constant heat flux |
| $\mathbf{H}$ | = heat flow vector |
| $k$ | = conductivity of the material |
| $L$ | = latent heat of the material |
| $q_1(t)$ | = unknown surface temperature |
| $q_2(t)$ | = melting distance |
| $t$ | = time |
| $\alpha$ | = thermal diffusivity |
| $\eta$ | = dimensionless melting distance = $(F/\rho L)(q_2/\alpha)$ |
| $\theta$ | = temperature distribution |
| $\theta_s$ | = surface temperature |
| $\rho$ | = density of the material |
| $\tau$ | = dimensionless time = $(F/\rho L)^2(t/\alpha)$ |
| $\psi$ | = dimensionless surface temperature = $c\theta/\rho L$ |

### Introduction

THE Stefan problem, also called the phase-change problem, involves the determination of the unknown moving boundary and the temperature-time history in a semi-infinite solid. The problem is classified into the following categories: 1) temperature as the boundary condition, 2) heat flux as the boundary condition with melt removed, 3) heat flux as the boundary condition without removal of melt, 4) heat flux at the boundary generated aerodynamically, 5) heat flux at the boundary is given and the melt begins to vaporize after time $t_0$.

Except for the first three cases,[1] no exact solutions are available for the phase-change problem. Goodman[2] has used his heat balance integral method in solving all the cases of the problem and the results have been found in agreement with exact solutions wherever possible.

Biot's variational principle, which is based upon the concepts of irreversible thermodynamics, has been well recognized as an

Received January 7, 1971; revision received October 26, 1971.
Index categories: Material Ablation; Heat Conduction.

* Lecturer, Department of Mechanical Engineering.

† Associate Professor, Department of Mechanical Engineering.